# Studies of group velocity reduction and pulse regeneration with and without the adiabatic approximation

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We present a detailed semiclassical study of the propagation of a pair of optical fields in resonant media with and without adiabatic approximation. In the case of near and on resonance excitation, we show detailed calculations, both analytically and numerically, on the extremely slowly propagating probe pulse and the subsequent regeneration of a pulse via a second pulse from the coupling laser. Further discussions on the adiabatic approximation provide many subtle details of the processes, including the effect on the pulse width of the regenerated optical field. Indeed, all features of the optical pulse regeneration and most of the intricate details of the process can be obtained with the present treatment without invoking a full field theoretical method. For very far off resonance excitation, we show that the analytical solution is nearly detuning independent, a surprising result that is vigorously tested and compared to numerical calculations with very good agreement.

## I. INTRODUCTION

Propagation of optical pulses in a resonant medium has long been an active field of research. The advances of laser technologies have stimulated further development and understanding of the propagation of optical pulses under various conditions, in particular in the field of ultra-short and extremely high intensity optical pulse generation and propagation under adiabatic conditions. Early works in this field include the interpretation of the nonlinear wave mixing in terms of an adiabatic following model [1], counter-intuitive pulse sequence [2] and induced transparency [3]. Later, Eberly and co-workers [4] developed a theory for the propagation of optical field under the condition that atomic response can be treated adiabatically. Further development on the propagation properties of a pulse pair under the similar adiabatical conditions resulted in substantial understanding of the evolution of optical pulses in a resonant media. In particular, Grobe and co-workers first proposed the creation and recall of excitations distributed in media [5]. Recently, the field has experience a renewed interest and resurgence following the demonstration of extremely slow propagation of optical pulse both in ultra-cold and high temperature atomic gases [6]. Experimental studies on possible excitation of the atomic spin wave excitations with ultra-slow optical field, and the subsequent "revival" of an optical field with frequencies near that of the ultra-slow field have promised future applications in information technology [7-9]. Theoretical works on such spin wave excitations and the subsequent "revival" of an optical filed have also been formulated in a "dark-state polariton" theory [10], which basically is a field theoretical reformulation of the adiabatic approximation of pulse pair propagation in a resonant medium originally formulated before [4,5]. These new treatments have provided some further understanding of the process, especially from the view point of atomic spin wave excitation and their relation to atomic coherence.

Here, we present a study on the propagation and "retrieval" of ultra-slow optical fields. We investigate both analytically and numerically the propagation problem, with and without adiabatic approximation, for the cases of near resonance, on-resonance, and far detuned from resonance. In all cases, we show that the slow group velocity propagation and pulse "revival" is independent of the one-photon detuning by the probe laser, providing that the probe and coupling lasers are tuned to exact two-photon resonance and certain conditions are satisfied. That is, the results on off-resonance excitations are identical to the adiabatic theory formulation for the exact on-resonance case treated in [4]. The main contribution of our work, however, is the detailed analysis, both analytical and numerical, on the extremely slow propagation of the optical field, the corresponding atomic response, and the "revival" of an optical field when a second time-delayed coupling pulse is injected into the medium. These results, to the best of our knowledge, are not contained in the original studies by Eberly and co-workers and all the subsequent studies, including all recent theoretical works, on the subject. In addition, we show that all the experimental observables can be well predicted with the usual treatment combining the classical electrodynamics and the three-state model, without invoking a full field theoretical methodology. This is not surprising, since when a detailed handling of spontaneous emission is not critical and the fields are not extremely weak, a mean-field approximation to the quantum treatment of the electromagnetic field is valid. In Section II, we first present an adiabatic theory for the case where non-vanishing one-photon detuning exists. This treatment is very similar to the original treatment given in [4] except that the latter deals exclusively with the case of on resonance excitation. In Section III, we examine the propagation of a pulse pair and give two examples on how to obtain the evolution of the optical fields. Section IV constitutes the main contribution of the present study. We discuss in detail the propagation of the pulse pair in the adiabatic limit and deduce analytically substantial insight and subtle understanding on the slow propagation of the probe field, the optical field-atomic spin cohenrence conversion, and the characteristics of the "retrieved" optical field. These results are not available in any of previous studies, and we believe they are very useful in providing a complete picture for the physics involved. In Section V, we compare predictions and estimates derived in Section IV with numerical simulations. Section VI is devoted to the case where the probe laser is tuned far from resonance, i.e. a Raman excitation scheme is employed. In this case, a different set of conditions are required for the adiabatic theory to be accurate. We show, however, that when certain conditions are satisfied, the solution is independent of the detuning, a surprising result that does not, to our best knowledge, exist in literature. Finally, the results are compared with numerical calculations for vigorous testing. In Section VII, we present a conclusion for our study.

## II. PRELIMINARY

In this section, we examine the propagation of a pair of optical pulse in resonant media under the conditions where the adiabatic approximation can be made. We present here an expanded investigation based on the early treatment given in [4]. The difference is the inclusion of the non-vanishing one-photon detuning since the original treatment assumed exact on one-photon resonance excitations. We will show that this non-vanishing one-photon detuning leads to correct predictions about pulse propagation, coherent population transfer and storage, and the regeneration of an optical field when a time delayed second pulse is provided from the coupling laser. It should be pointed out that all theoretical studies published recently on this subject also assume exact on resonance tuning by the probe laser as originally assumed in Ref.[4]. Readers should consult a series pioneer studies on the on-resonance case given in [4].

Consider a $\Lambda$ system as depicted in Figure 1. We assume that a probe laser ($E_p$, frequency $\omega_p$) is tuned on or near resonance with the $|1>\to|2>$ transition, and a coupling laser ($E_c$, frequency $\omega_c$) is tuned so that exact two-photon resonance between states $|1>$ and $|3>$ is achieved. Assume atomic wave function of the form

$$|\Psi(z,t_r)>= a_1 e^{-i\omega_1 t}|1> + a_2 e^{-i\omega_2 t}|2> + a_3 e^{-i\omega_3 t}|3>, \tag{1}$$

we thus find the following atomic equations of motion

$$\frac{\partial a_1}{\partial t_r} = i\Omega_p e^{-ik_p z} a_2, \tag{2a}$$

$$\frac{\partial a_2}{\partial t_r} = i\Omega_p^* e^{ik_p z} a_1 + i\Omega_c^* e^{ik_c z} a_3 + i\left(\delta + i\frac{\gamma_2}{2}\right) a_2, \tag{2b}$$

$$\frac{\partial a_3}{\partial t_r} = i\Omega_c e^{-ik_c z} a_2, \tag{2c}$$

where $a_j$ and $\gamma_j$ are the $jth$ amplitude of the atomic wave function and decay rate, respectively. $\Omega_p$ and $\Omega_c$ are half-Rabi frequencies, given by $\Omega_p^* = D_{21}E_p/(2\hbar)$, $\Omega_c^* = D_{23}E_c/(2\hbar)$ with $D_{ij}$ being the dipole moment of the relevant transition. Also, $\omega_p = \omega_2 - \omega_1 + \delta$ $\omega_c = \omega_2 - \omega_3$, and the quantity $t_r = t - z/c$ is the retarded time which is the combination of $t$ and $z$ that the laser field amplitudes depend on in vacuum.

In order to correctly describe the propagation of the optical pulse, the atomic equations of motion (2) must be simultaneously solved self-consistently with Maxwell equations. In the limit of plane waves and slowly varying amplitudes, the positive frequency part of these fields satisfy

$$\left(\frac{\partial \Omega_p^*}{\partial z}\right)_{t_r} = i\kappa_{12}\tau A_1^* A_2, \tag{3a}$$

$$\left(\frac{\partial \Omega_c^*}{\partial z}\right)_{t_r} = i\kappa_{32}\tau A_3^* A_2, \tag{3b}$$

where the amplitudes $A_1$, $A_2$, and $A_3$ differ by position dependent phase factors from $a_1$, $a_2$, and $a_3$, and we have introduced notations $\kappa_{12} = 2\pi N\omega_p|D_{12}|^2/(\hbar c)$ and $\kappa_{32} = 2\pi N\omega_c|D_{32}|^2/(\hbar c)$ with $N$ being the concentration.

In order to evaluate the polarization terms on the right hand sides of Eq.(3a,3b), it is necessary to obtain the amplitudes of the atomic wavefunction. The problem is intrinsically difficult because both Eq.(2) and (3) must be solved simultaneously. A great simplification can be achieved if one invokes the adiabatic approximation while evaluating the atomic wavefunction. The wavefunction obtained with the adiabatic limit will then be used to calculate the optical field in a self-consistent manner. This is the spirit of all adiabatic approximation based optical propagation problem and will be the method used in the present work.

In an adiabatic treatment we introduce $A_1 = a_1$, $A_2 = e^{ik_p z}a_2$, and $A_3 = e^{ik_c z}a_3$ in order to remove the $z$ dependent phase factors. The adiabatic eigenvalues of the equations of motion can then be obtained from the characteristic equation given by

$$\lambda^3 - \left(\delta + i\frac{\gamma_2}{2}\right)\lambda^2 - (|\Omega_p|^2 + |\Omega_c|^2)\lambda = 0.$$

They are

$$\lambda_0 = 0, \tag{4a}$$

$$\lambda_+ = \left(\frac{\delta + i\frac{\gamma_2}{2}}{2}\right) + \sqrt{|\Omega_p|^2 + |\Omega_c|^2 + \left(\frac{\delta + i\frac{\gamma_2}{2}}{2}\right)^2}, \tag{4b}$$

$$\lambda_- = \left(\frac{\delta + i\frac{\gamma_2}{2}}{2}\right) - \sqrt{|\Omega_p|^2 + |\Omega_c|^2 + \left(\frac{\delta + i\frac{\gamma_2}{2}}{2}\right)^2}. \tag{4c}$$

As can be seen, the inclusion of the non-vanishing one-photon detuning changes the eigenvalues. For sufficiently small $\delta$, this condition reduces correctly to that given in [4].

The adiabatic condition requires that the eigenvalue $\lambda_0\tau$ always differ from $\lambda_\pm\tau$ by an amount that is large compared with unity. When $|\delta\tau| >> |\Omega_c|$ and $|\delta\tau| >> |\Omega_p|$, however, additional conditions are required. In fact, we require that the pulse length of the coupling laser be much longer than that of the probe laser, and $|\Omega_c\tau|^2/|\delta\tau| >> 1$ be satisfied. Therefore, during the entire pulse of the probe laser, the two-photon transition between $|1>$ and $|3>$ is shifted well outside the bandwidth of the probe laser. In another words, the overlap of the probe pulse band width with the resonance must be avoided.

We now focus on the eigenvector corresponding to $\lambda_0 = 0$, since it is this adiabatic state that corresponds to the population all being in state $|1>$ before the laser pulses. In the lowest order of the adiabatic approximation Eqs.(2a) and (2c) imply that $a_2 = 0$. Eq.(2b) then yields

$$A_1 = -\frac{\Omega_c^*}{\Omega_p^*}A_3. \tag{5}$$

Since, for a closed system, $|a_1|^2 + |a_2|^2 + |a_3|^2 = 1$ we can write

$$A_1(z, t_r) = \frac{\Omega_c^*(z, t_r)}{\sqrt{|\Omega_c(z, t_r)|^2 + |\Omega_p(z, t_r)|^2}}, \tag{6a}$$

$$A_2(z, t_r) = 0, \tag{6b}$$

$$A_3(z, t_r) = -\frac{\Omega_p^*(z, t_r)}{\sqrt{|\Omega_c(z, t_r)|^2 + |\Omega_p(z, t_r)|^2}}. \tag{6c}$$

It is important to realize that solutions (6a-6c) are not sufficiently accurate for many propagation problems. This is because that even though numerical solutions show that $A_2$ is much smaller than the amplitudes of the other states, it cannot be exactly zero without assuming the laser pulses are propagating at exactly the vacuum speed of light. In many situations, however, this assumption of the speed of laser pulses is not valid. This is particularly true in the case of sufficiently high optical density and very near a strong resonance excitation where significant modification of the propagation velocity is anticipated. In order to accurately account the propagation effect in the regime where significant dispersion effect is encountered, one must seek the next order correction to the lowest order approximation where the small quantities such as $A_2$ becomes critically important. To do this, one can use either Eq.(2a) or Eq.(2c) to derive a second approximation for $A_2$, thereby obtains the first order correction to the lowest order adiabatic approximation. Substitute Eq.(6a) into Eq.(2a) or Eq.(6c) into Eq.(2c) we obtain

$$A_2(z, t_r) = -\frac{i}{\Omega_p}\frac{\partial A_1}{\partial t_r} = -\frac{i}{\Omega_p}\frac{\partial}{\partial t_r}\left(\frac{\Omega_c^*}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}}\right), \tag{7a}$$

and

$$A_2(z, t_r) = -\frac{i}{\Omega_c}\frac{\partial A_3}{\partial t_r} = \frac{i}{\Omega_c}\frac{\partial}{\partial t_r}\left(\frac{\Omega_p^*}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}}\right). \tag{7b}$$

Within the adiabatic approximation and the slowly varying amplitude approximation one can show that $\Omega_p(z,t_r)$ and $\Omega_c(z,t_r)$ can be chosen to be real. It is then trivial to show that two forms for $A_2$ as shown in Eqs.(7a,7b) are self consistent. It is interesting to note that even though having $\delta$ non-zero makes the criteria for the validity of the adiabatic approximation different, it does not change the solution for the state amplitudes.

As usual with the adiabatic approximation used in a resonance situation, we must have $\Omega_c$ already strong when $\Omega_p$ starts to build up. This means that $\Omega_c\tau >> 1$, where $\tau$ is a measure of the time over which $\Omega_c$, or $\Omega_p$ change significantly. One scenario where the adiabatic approximation would hold through the whole laser pulse is if the two lasers peak at the same time, but the pulse length of the coupling laser is much longer. This feature of the pulse lengths and the requirement $|\Omega_c\tau| >> 1$ through the pulse length of the probe laser will make results based on the adiabatic approximation quite accurate. In essence, one must keep both $|\lambda_+\tau| >> 1$ and $|\lambda_-\tau| >> 1$ throughout the pulse of the probe laser, e.g. there is a wide gap between $\lambda_0 = 0$ and the other eigenvalues so that curve crossing is avoided throughout the entire pulse length. We will now investigate the use of the adiabatic approximation in the propagation problem.

## III. THE PROPAGATION PROBLEM

In order to treat the propagation of the laser pulses through a resonant medium, the polarization of the medium as a function of position and time must be obtained. Using the atomic wave function obtained in Sec. II, we get

$$\begin{aligned} P(t) &= N < \Psi(z,t_r)|\hat{D}|\Psi(z,t) > \\ &= A_1^*(z,t_r)A_2(z,t_r)e^{ik_pz-i\omega_pt}ND_{12} + A_3^*(z,t_r)A_2(z,t_r)e^{ik_cz-i\omega_ct}ND_{32} + c.c. \end{aligned} \tag{8}$$

where $N$ is the concentration. For the positive frequency parts of the polarizations at the probe and coupling laser frequencies, we have

$$P^+_{\omega_p} = ND_{12}e^{ik_p-i\omega_pt}A_1^*A_2, \tag{9a}$$

$$P^+_{\omega_c} = ND_{32}e^{ik_c-i\omega_ct}A_3^*A_2. \tag{9b}$$

Using the first order correction to $A_2$, i.e. Eqs.(7a,7b), we find, for the amplitude of $P^+_\omega$

$$P_{\omega_p,0} = iND_{12}\frac{1}{\sqrt{|\Omega_c|^2+|\Omega_p|^2}}\frac{\partial}{\partial t_r}\left(\frac{\Omega_p^*}{\sqrt{|\Omega_c|^2+|\Omega_p|^2}}\right), \tag{10a}$$

$$P_{\omega_c,0} = iND_{32}\frac{1}{\sqrt{|\Omega_c|^2+|\Omega_p|^2}}\frac{\partial}{\partial t_r}\left(\frac{\Omega_c^*}{\sqrt{|\Omega_c|^2+|\Omega_p|^2}}\right). \tag{10b}$$

Substitute these polarizations into Eq.(3a,3b), we obtain

$$\left(\frac{\partial\Omega_p^*}{\partial z}\right)_{t_r} = -\frac{\kappa_{12}\tau}{\sqrt{|\tau\Omega_c|^2+|\tau\Omega_p|^2}}\frac{\partial}{\partial t_r/\tau}\left(\frac{\tau\Omega_p^*}{\sqrt{|\tau\Omega_c|^2+|\tau\Omega_p|^2}}\right), \tag{11a}$$

$$\left(\frac{\partial\Omega_c^*}{\partial z}\right)_{t_r} = -\frac{\kappa_{32}\tau}{\sqrt{|\tau\Omega_c|^2+|\tau\Omega_p|^2}}\frac{\partial}{\partial t_r/\tau}\left(\frac{\tau\Omega_c^*}{\sqrt{|\tau\Omega_c|^2+|\tau\Omega_p|^2}}\right). \tag{11b}$$

Equations (11) are the key equations describing propagation of a pulse pair within the adiabatic approximation. By using the adiabatic approximation for the atomic response one needs to solve only two nonlinear partial differential equations, instead of solving five nonlinear partial differential equations Eq.(2a-2c) and Eq.(3a,3b).

To further illustrate the effectiveness of the adiabatic approximation, we now examine two special cases.

We proceed by first noticing that the quantity

$$F = \frac{|\Omega_p|^2}{\kappa_{12}} + \frac{|\Omega_c|^2}{\kappa_{32}}, \tag{12}$$

represents the sum of the photon fluxes at $\omega_p$ and $\omega_c$ divided by the concentration of the medium through which the waves propagate. Differentiate $F$ with respect to $z$ while holding $t_r$ fixed and make use of Eq.(11), we obtain

$$\frac{\partial}{\partial z}\left(\frac{|\Omega_p|^2}{\kappa_{12}} + \frac{|\Omega_c|^2}{\kappa_{32}}\right) = 0. \tag{13}$$

Thus, $F$ does not depends explicitly on $z$. This permits us to evaluate $F$ by evaluating it at $z = 0$, the entrance to the atomic vapor cell, e.g.

$$F(z,t) = F(z=0,t) = \frac{|\Omega_p(0,t)|^2}{\kappa_{12}} + \frac{|\Omega_c(0,t)|^2}{\kappa_{32}}. \tag{14}$$

Thus, whenever $|\Omega_c(z,t_r)|^2/\kappa_{32} + |\Omega_p(z,t_r)|^2/\kappa_{12}$ occurs it can be replaced by $F(t_r)$, as determined in Eq.(14). An exact relation for this quantity can also be directly derived from Eq.(2a-2c) and Eq.(3a,3b). One obtains

$$\frac{\partial}{\partial z}\left(\frac{|\Omega_p|^2}{\kappa_{12}} + \frac{|\Omega_c|^2}{\kappa_{32}}\right) = -\frac{\partial |A_2|^2}{\partial t_r} - \gamma_2 |A_2|^2,$$

This relation indicates that anytime $|A_2|^2$ is very small and slowly varying, and the decay rate of this state is not too fast, this total photon flux is close to depending only on $t_r$. The lack of dependency of this quantity on $z$ when the full set of equations is solved numerically is an important test for the validity of the adiabatic approximation.

### Case 1. $\kappa_{12} = \kappa_{32}$

In this situation the quantity $|\Omega_p(z,t_r)|^2 + |\Omega_c(z,t_r)|^2 = \kappa_{12}F(t_r)$. We now use this fact in Eqs.(11a,b) to obtain an analytical approximation that can be compared with numerical solutions to Eqs.(2a-2c) and (11a,b).

Since $\sqrt{|\Omega_c|^2 + |\Omega_p|^2}$ does not depend on $z$ when $t_r$ is held fixed, dividing both sides of Eqs.(11a,b) by this quantity yields

$$\frac{\partial}{\partial z}\left(\frac{\Omega_p^*\tau}{\sqrt{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}}\right) = -\frac{\kappa_{12}\tau}{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}\frac{\partial}{\partial t_r/\tau}\left(\frac{\Omega_p^*\tau}{\sqrt{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}}\right), \tag{15a}$$

$$\frac{\partial}{\partial z}\left(\frac{\Omega_c^*\tau}{\sqrt{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}}\right) = -\frac{\kappa_{12}\tau}{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}\frac{\partial}{\partial t_r/\tau}\left(\frac{\Omega_c^*\tau}{\sqrt{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}}\right). \tag{15b}$$

A few substitutions make an analytical solution to these equations relatively obvious. Let

$$W_p = \frac{\Omega_p^*\tau}{\sqrt{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}}, \tag{16a}$$

$$W_c = \frac{\Omega_c^*\tau}{\sqrt{|\Omega_c\tau|^2 + |\Omega_p\tau|^2}}, \tag{16b}$$

$$v(t_r) = \int_{-\infty}^{t_r/\tau}\left(|\Omega_c(0,t_r^{'})\tau|^2 + |\Omega_p(0,t_r^{'})\tau|^2\right) d\left(\frac{t_r^{'}}{\tau}\right), \tag{16c}$$

$$u(z) = \int_0^z \kappa_{12}\tau dz^{'}. \tag{16d}$$

In terms of these dimensionless quantities Eqs.(15) become

$$\frac{\partial W_p}{\partial u} + \frac{\partial W_p}{\partial v} = 0, \tag{17a}$$

$$\frac{\partial W_c}{\partial u} + \frac{\partial W_c}{\partial v} = 0, \tag{17b}$$

where general “travelling wave” type solutions are immediately obtained as

$$W_p = F_p(v-u), \tag{18a}$$

$$W_c = F_c(v-u). \tag{18b}$$

This special case provides a very useful test case for examining the validity of the adiabatic approximation for the "trapped" light problem. The functions $F_p$ and $F_c$ are easily determined by evaluating at $z = 0$ (therefore, $u = 0$) and using the fact that the laser fields are known as a function of $t$ and hence $v$. When the second coupling laser pulse is sent into the medium after a time delay, the predictions about the "revival" of the probe laser are all contained in this solution. The tabulation of $F_p$ remains the same as long as only a second coupling laser is sent into the medium after some delay time.

**Case 2.** $|\Omega_p(0,t)\tau| << |\Omega_c(0,t)\tau|$ **with** $\Omega_p\tau << 1$

In this limit there can never be a significant population in state $|3>$. Therefore, to a good approximation we have $\Omega_c(z,t_r) = \Omega_c(0,t_r)$. That is, the coupling laser is not effected very much by the resonant medium and it propagates at speed $c$ without distortion. Thus, in this limit we only need to solve Eq.(11a) which becomes

$$\frac{\partial}{\partial z}\left(\frac{\Omega_p^*\tau}{|\Omega_c(0,t_r)\tau|}\right) = -\frac{\kappa_{12}\tau}{|\Omega_c(0,t_r)\tau|^2}\frac{\partial}{\partial t_r/\tau}\left(\frac{\Omega_p^*\tau}{|\Omega_c(0,t_r)\tau|}\right). \tag{19}$$

If we make the following dimensionless substitutions

$$W_p = \frac{\Omega_p^*}{|\Omega_c(0,t_r)|}, \tag{20a}$$

$$v(t_r) = \int_{-\infty}^{t_r/\tau}\left(|\Omega_c(0,t_r^{'})\tau|^2\right)d\left(\frac{t_r^{'}}{\tau}\right), \tag{20b}$$

$$u(z) = \int_0^z \kappa_{12}\tau dz^{'}, \tag{20c}$$

we then find that the solution for $W_p$ is

$$W_p = F_p(v-u), \tag{21}$$

with the function $F_p$ being determined by using $z = 0$ at the entrance of the cell,

$$F_p(v(t)) = \frac{\Omega_p(0,t)}{|\Omega_c(0,t)|}, \tag{22}$$

where $v(t)$ is given in Eq.(20b), with $t_r = t$ at $z = 0$. A table of $F_p$ as a function of its argument can be made, which will be used when $z$ is no longer zero and the argument is $v(t_r) - u(z)$. Once we have tabulated $F_p$, the probe field at different $z$ can be trivially calculated using the known coupling field, e.g.

$$\Omega_p^*(z,t_r) = |\Omega_c(0,t_r)|F_p(v(t_r) - u(z)). \tag{23}$$

Again, the tabulation of $F_p$ is determined completely at all later times by Eq (22) - at least for the case where only a time delayed coupling laser pulse enters the material at $z = 0$.

## IV. ADIABATIC APPROXIMATION APPLIED TO OPTICAL FIELD REGENERATION

In the precious sections, we have provided detailed descriptions and examples on how adiabatic approximation works. In this section, we will discuss various aspects of the adiabatic approximation, especially the validity in the context of probe pulse "revival". These discussions provide many subtle understandings on the process which, in our view, are not discussed either in the recent works on "probe pulse revival" and the original studies of adiabatic approximation. This section, therefore, constitutes the main contributions of the present study.

The validity of the adiabatic approximation depends on more than just the slow time dependence of the amplitudes of the probe and coupling lasers. For instance, the approximation will fail in dealing with the solution for the atomic response if the two pulses are sent simultaneously and the pulse widths and shapes are the same, even with both fields being turned on slowly. In this situation, at early times substantial populations will be promoted to state $|2>$, in contradiction to the adiabatic solution which, to the lowest order, predicts a zero population for this state. What is needed is to have the coupling laser built up and establish laser induced transparency before the probe laser starts to become intense. This could be done by either time delaying the probe laser relative to the coupling laser, or by making the coupling laser pulse length much longer than that of the probe laser and by having them peak in intensity at the same time. In the following treatment we will consider the latter situation. We therefore will choose the characteristics of $\Omega_p(0,t)$ and $\Omega_c(0,t)$ that would be conducive to making the adiabatic approximation work well, and demonstrate that this is indeed the case. We emphasize that our choice of the form of the laser fields is only one of many workable assumptions on the acceptable forms of the laser pulses that will make the adiabatic theory work well.

Let us consider the functional forms for $\Omega_p(0,t)$ and $\Omega_c(0,t)$ as

$$\Omega_p(0,t) = \Omega_{p0} e^{-(t/\tau)^2}, \tag{24a}$$

$$\Omega_c(0,t) = \Omega_{c0} \left( e^{-0.2(t/\tau)^2} + R e^{-0.2(t/\tau - x_0)^2} \right). \tag{24b}$$

With this functional form, the Rabi frequency due to the coupling laser rises up to produce laser induced transparency before the probe laser becomes large enough to have any effect, an important consequence of having a significantly longer pulse length for the coupling laser. Without this characteristic of the coupling laser, the probe laser would produce a population in state $|2>$ before the transparency was induced. The production of population in $|2>$ would mean that different adiabatic states have already been mixed and we have a failure of a strictly adiabatic picture in which only a single adiabatic state persists throughout the pulse. In Eq.(24), $\Omega_{p0}$ and $\Omega_{c0}$ are real constants characterising the peak amplitude of the two half-Rabi frequencies before the pulses enter the resonant medium. The parameter $\tau$ is a measure of the pulse length of the probe laser, $R$ is the ratio of the Rabi frequency at which the coupling laser recurs to its initial amplitude, $x_0 = t_d/\tau$ is the value of $t_r/\tau$ at which the peak of the coupling laser recurs.

We will begin by pointing out how the adiabatic approximation can be used to understand what happens for $R = 0$. That is, for the case where there is no recurring coupling laser pulse. We take $\Omega_{p0}\tau = \Omega_{c0}\tau = 20$, $\kappa_{12}\tau = \kappa_{32}\tau = 700 cm^{-1}$, and $\gamma_2\tau = 0$. We will soon see that in this numerical example the group velocity is sufficiently small so that the coupling laser dies away before the probe pulse can propagate through the vapor cell. When the coupling laser begins to die out, the intensity of the two lasers becomes proportional to each other. This is because when the coupling laser is beginning to become small, the following relation is appropriate (see Eq.(16c))

$$v(t_r) \simeq |\Omega_{c0}\tau|^2 \sqrt{5\pi/2} + |\Omega_{p0}\tau|^2 \sqrt{\pi/2}. \tag{25}$$

That is, almost all of the area under the two Gaussian pulse shapes has already been included in the integration. Also, $|\Omega_p(0,t_r)|^2$ has long been very small compared with $|\Omega_c(0,t_r)|^2$. Thus, for $t_r/\tau > 3.0$ we have, as a very good approximation

$$\Omega_p^*(z,t_r) \simeq \sqrt{|\Omega_c(0,t_r)|^2 + |\Omega_p(0,t_r)|^2} F_p((|\Omega_{c0}\tau|^2 \sqrt{5\pi/2} + |\Omega_{p0}\tau|^2 \sqrt{\pi/2}) - \kappa_{12}\tau z). \tag{26}$$

(In the case of an inhomogeneous medium $\kappa_{12}\tau z$ would be replaced by an integral over the $z$ dependent $\kappa_{12}\tau$.) At such a late time during the pulse, since the argument of $F_p$ depends only on the $z$ coordinate, therefore the time dependence of $\Omega_p^*$ is obviously exactly the same as that of $\sqrt{|\Omega_c(0,t_r)\tau|^2 + |\Omega_p(0,t_r)\tau|^2}$. When $|\Omega_c(0,t_r)|$ is several times larger than $|\Omega_p(0,t_r)|$, as always is at such late times at $z = 0$, this means that $\Omega_p^*(z,t_r)$ has the same time dependence as $\Omega_c^*(0,t_r)$ at such late times. The fact that the two pulses have the same time dependence as they begin to become very weak is the reason that populations are left in each of the states $|1>$ and $|3>$. If one of the laser fields died out much more rapidly than the other there would only be population left in only one state.

¿From Eq.(26) we see that if we choose $z$ such that

$$\kappa_{12}\tau z = \frac{1}{2} \left( |\Omega_{c0}\tau|^2 \sqrt{5\pi/2} + \sqrt{\pi/2} |\Omega_{p0}\tau|^2 \right),$$

then the argument of $F_p$ will be the same as at $z = 0$ and $t_r = t = 0$. With this argument, the value of $F_p$ is $F_p = \Omega_{p0}/\sqrt{|\Omega_{c0}|^2 + |\Omega_{p0}|^2}$. This is the largest value that $F_p$ takes on. Thus, at this depth into the medium the value of $|A_3(z,t_r)|$ matches its largest value at $z = 0$. This population persists at large $t_r$ until very slow collisional effects either destroy the coherence left behind in states $|1>$ and $|3>$, or until collisions mix the populations of the two states. This long persistence of a coherent mix of populations in states $|1>$ and $|3>$ is what leads to the "revival" of an optical field with frequency very close to that of the original probe laser when a coupling laser is re-injected into the medium after some time delay [11]. The number of atoms left in $|3>$ is (within the adiabatic approximation) equal to the number of photons in the original probe pulse (recall that we have chosen parameters so that the probe pulse does not penetrate the entire medium). The creation of a coherent mixed stat is also what makes this situation interesting from the point of view of quantum computing.

Next, we investigate the situation where $R > 0$, i.e. a second coupling pulse enters the medium at a delayed time. Let $\Omega_{c0}\tau = 20$, $\Omega_{p0}\tau = 5$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$. From the above consideration, we expect $|A_3|$ to have its largest value at large $t_r$ when $z = 2.86$ cm. In Figure 2 we show a contour plot of $|A_3(z, t_r)|$ based on our adiabatic theory. The predicted $|A_3|$ is indicated by the line of constant color leading from $t_r = 0$ and $z = 0$ out to the horizontal path at $z \simeq 2.86$ cm, as expected. In Figure 3 we show a surface plot of the probe field Rabi frequency $\Omega_p\tau$ as functions of $t_r/\tau$ at $z = 3$ cm. The long asymmetric tail of $\Omega_p$ is a consequence of the behavior described in Eq.(26). In the region between $2.5 \leq t_r/\tau \leq 5$ the ratio of the two half-Rabi frequencies is close to constant, averaging around 0.24. This ratio is also close to the adiabatic approximation for $A_3$ since $|\Omega_p|^2 << |\Omega_c|^2$. In a different set of parameters, we choose $\Omega_{c0}\tau = \Omega_{p0}\tau = 20$, $\gamma_2\tau = 0$, $R = 2.923$, and $\kappa_{12}\tau = \kappa_{32}\tau = 700cm^{-1}$. In this example,our theory predicts that the peak value of $|A_3(t_r, z)|$ before the second coupling laser pulse enters the medium will occur at $z = 1.159$ cm. Indeed, as can be seen clearly from Figure 4 that $|A_3(z, t_r)|$ takes on the value $1/\sqrt{2}$ at a depth of about 1.1-1.2 cm into the medium, as predicted. Figure 5 depicts the corresponding probe field predicted with adiabatic theory for this set of parameters.

We now illustrate how one can take a set of initial conditions and very simply determine the characteristics of the "revived" probe pulse at the point it emerges from the atomic vapor.

First, note that with our choice of pulse characteristics and during the recurring coupling laser pulse

$$v(t_r) = S + \frac{R^2}{2}|\Omega_{c0}\tau|^2\sqrt{\frac{5\pi}{2}}\left(1 + erf\left(\sqrt{\frac{2}{5}}\frac{(t_r - t_d)}{\tau}\right)\right), \tag{27a}$$

$$S = |\Omega_{c0}\tau|^2\sqrt{5\pi/2} + |\Omega_{p0}\tau|^2\sqrt{\pi/2}. \tag{27b}$$

The reader will recall that $F_p$ was determined from the functional dependence of the probe and coupling laser pulses at $z = 0$. In particular, $W_p(v(t)) = -A_3(0, t)$, where $A_3(0, t)$ is determined from Eq.(6c) and the pulse characteristics at $z = 0$. In addition, the following properties of the function $F_p$ are very useful:

$$F_p(0) = 0, \tag{28a}$$

$$F_p(S) = 0, \tag{28b}$$

$$F_p(S/2) = F_p(v(0)) = \frac{\Omega_{p0}\tau}{\sqrt{|\Omega_{c0}\tau|^2 + |\Omega_{p0}\tau|^2}}. \tag{28c}$$

As will be seen later, we will invoke these properties when $z = z_m$ is reached, where $z_m$ is the value of $z$ at the end of the medium, to estimate the time interval over which the "revived" probe pulse exits, as well as the peak amplitude of $\Omega_p^*(z_m, t_r)$.

Before proceeding further, we will point out some obvious properties that must be satisfied by the parameter, $R$, that appears in Eq.(24b). We first note that $R$ must be large enough so that $v(t_r) - \kappa_{12}\tau z_m > 0$. That is, $R$ must be large enough so that the group velocity becomes large enough to allow most of the regenerated photons to exit the cell before the laser induced transparency ends. This group velocity depends on position and time as (see Eq.(15))

$$\frac{c}{v_g} = 1 + \frac{\kappa_{12}c\tau^2}{|\Omega_p(z, t_r)\tau|^2 + |\Omega_c(z, t_r)\tau|^2}. \tag{29}$$

We first write down the condition for a time $t_{r1}$ at which the "revived probe pulse" first reaches $z_m$. This is the earliest time at which the argument of $F_p$ goes from being negative and becomes zero. At this point $F_p(0) = 0$, but at later times it will become non-zero. This time is determined by

$$v(t_{r1}) - \kappa_{12}\tau z_m = 0. \tag{30}$$

At time $t_{rm}$ the value of $F_p(S/2)$ will be equal to $-A_3(0, 0)$. This is the maximum value $F_p$ takes on with the set of parameters chosen. This time is determined by

$$v(t_{rm}) - \kappa_{12}\tau z_m = S/2. \tag{31}$$

Finally, at the time $t_{r2}$ at which the "revived probe pulse" completes its exit from the medium. At this time, we have $F_p(S) = 0$. Therefore,

$$v(t_{r2}) - \kappa_{12}\tau z_m = S. \tag{32}$$

Using Eqs.(27a,b) we can rewrite Eqs.(30-32) as

$$erf\left(\sqrt{2/5}\frac{(t_{r1}-t_d)}{\tau}\right) = \frac{2(\kappa_{12}\tau z_m - S)}{R^2|\Omega_{c0}\tau|^2\sqrt{5\pi/2}} - 1, \tag{33a}$$

$$erf\left(\sqrt{2/5}\frac{(t_{rm}-t_d)}{\tau}\right) = \frac{2(\kappa_{12}\tau z_m - S/2)}{R^2|\Omega_{c0}\tau|^2\sqrt{5\pi/2}} - 1. \tag{33b}$$

$$erf\left(\sqrt{2/5}\frac{(t_{r2}-t_d)}{\tau}\right) = \frac{2(\kappa_{12}\tau z_m)}{R^2|\Omega_{c0}\tau|^2\sqrt{5\pi/2}} - 1, \tag{33c}$$

which can be further condensed into

$$erf\left(\sqrt{2/5}\frac{(t_r-t_d)}{\tau}\right) = \frac{\alpha - f\beta}{R^2} - 1,$$

where

$$\alpha = \frac{2\kappa_{12}\tau z_m}{|\Omega_{c0}\tau|^2\sqrt{5\pi/2}},$$
$$\beta = \frac{2S}{|\Omega_{c0}\tau|^2\sqrt{5\pi/2}},$$

with $f = 1$ to determine $t_{r1}$, $f = 0.5$ to determine $t_{rm}$, and $f = 0$ to determine $t_{r2}$. Correspondingly, we obtain the peak value of $\Omega_p^*(z_m, t_{rm})\tau$ as

$$\Omega_p^*(z_m, t_{rm}) = R\Omega_{c0}e^{-(t_{rm}-t_d)^2/(5\tau^2)}\frac{\Omega_{p0}\tau}{\sqrt{|\Omega_{c0}\tau|^2 + |\Omega_{p0}\tau|^2}}. \tag{34}$$

As an example, let $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $z_m = 8$ cm, $\Omega_{c0}\tau = 20$, $\Omega_{p0}\tau = 5$, $R = 4$, and $\gamma_2\tau = 0$. With these parameters, we find $\alpha = 2.854598$, $\alpha/R^2 = 0.1784124$, $\beta = 2.0559017$, and $\beta/R^2 = 0.128494$. Thus, $(t_{r1} - t_d)/\tau = -2.19$, $(t_{rm} - t_d)/\tau = -1.767$, and $(t_{r2} - t_d)/\tau = -1.50$. The value of $\Omega_p^*(z_m, t_{rm})$ is thus estimated as

$$\Omega_p^*(z_m, t_{rm}) = 4(20)\exp(-0.2(1.767)^2)\frac{5}{\sqrt{20^2 + 5^2}} = 10.39.$$

The time $t_{rm}$ corresponds to the largest value of $F_p$, but not quite to the maximum of $\Omega_p^*(z_m, t_r)$. The actual maximum occurs closer to -1.74 with the maximum value of 10.44. However, our simple estimate usually does very well if $R$ is large enough to easily let all of the regenerated probe laser photons escape.

We next note that if $R$ is chosen to make the argument of $F_p$ exactly the same as its value at $z = 0$ and $t_r = t = 0$ at the time $t_r = t_d$, we then have

$$|\Omega_{p0}\tau|^2\sqrt{\frac{\pi}{2}} + |\Omega_{c0}\tau|^2\sqrt{\frac{5\pi}{2}}\left(1 + \frac{R^2}{2}\right) - \kappa_{12}\tau z_m = \frac{1}{2}\left(|\Omega_{c0}\tau|^2\sqrt{\frac{5\pi}{2}} + |\Omega_{p0}\tau|^2\sqrt{\frac{\pi}{2}}\right). \tag{35}$$

Thus,

$$|\Omega_{p0}\tau|^2\sqrt{\pi/8} + |\Omega_{c0}\tau|^2\sqrt{5\pi/8}(1 + R^2) = \kappa_{12}\tau z_m. \tag{36}$$

By choosing the value of $R$ such that this equation is satisfied, one achieves a situation where the peak value of $\Omega_p(z_m, t_d)$ is given by (see Eq.(34))

$$\Omega_{p0}^*(z_m, t_d) = R|\Omega_{c0}|\frac{\Omega_{p0}\tau}{\sqrt{|\Omega_{c0}\tau|^2 + |\Omega_{p0}\tau|^2}}. \tag{37}$$

The characteristics of this regenerated field can be estimated as follows. We first determine the value of $t_r$ such that the right hand side of Eq.(32) is zero or the full area under square of the incident probe and coupling lasers. This yields a range of time over which the probe rises from zero and returns to zero at the exit of the cell. One half of this time interval is close to the full-width at half-maximum of the exiting probe laser beam. We then note that our condition on $R$ yields

$$R^2 = \frac{\kappa_{12}\tau z_m - |\Omega_{c0}\tau|^2\sqrt{5\pi/8} - |\Omega_{p0}\tau|^2\sqrt{\pi/8}}{|\Omega_{c0}\tau|^2\sqrt{5\pi/8}}. \tag{38}$$

Using these considerations we estimate the full-width at half-maximum pulse length, in the unit of the original probe pulse length $\tau$, to be

$$\Delta_{1/2} = \frac{\sqrt{5\pi/2}}{R^2}\left(1 + \frac{1}{\sqrt{5}}\left|\frac{\Omega_{p0}}{\Omega_{c0}}\right|^2\right). \tag{39}$$

If the original probe laser has only penetrated a small fraction of the thickness of the medium when the coupling laser pulse has passed by, then $R$ will turn out to be much larger than unity. This means that the width of the regenerated field is generally much smaller than the width of the initial probe pulse. Correspondingly, the bandwidth of the light will be larger. An immediate conclusion is thus available: the regenerated pulse is not the replica of the original probe pulse. It is coherent, to an extended degree, with the coupling laser because of the stimulated nature of the regeneration process. Therefore, the notion of "coherent storage" of the information carried in the original probe field is not accurate [7-9]. These features can be seen from Figures 3 and 5 which show $\Omega_p^*$ as a function of $t_r/\tau$ and $z$, as well as the time dependence of the regenerated field at the exit of the medium. In particular, as can be clearly seen from Figures 3 and 5, there is no optical field left in the region between the time when the original probe pulse and the companion coupling pulse enter the medium and the time when the second coupling pulse enters the medium. Therefore, photons are neither "stored" nor "stopped". All photons from the original probe pulse are converted to the population of the state $|3>$. These figures also show that in "reviving the probe pulse" every regenerated photon comes at the expense of flipping population from state $|3>$ to state $|1>$. When the population of $|3>$ has been exhausted, there can be no further photon generated. Thus, in cases where the exiting regenerated field has a peak intensity much larger than the initial probe beam, the pulse width of this exiting pulse must be necessarily and correspondingly narrower. This is why the peak intensity of the regenerated field can go as $R^2$, while the width of the pulse tends to go as $1/R^2$.

## V. COMPARISON BETWEEN NUMERICS AND THE ADIABATIC THEORY

We will now present some examples showing the degree of agreement between numerical calculations and the adiabatic theory developed in the previous sections. To obtain the numerical results, we solve numerically the coupled Maxwell and time dependent Schrödinger equations

$$\left(\frac{\partial A_1}{\partial t_r}\right)_z = i\Omega_p A_2, \tag{40a}$$

$$\left(\frac{\partial A_2}{\partial t_r}\right)_z = i\Omega_p^* A_1 + i\Omega_c^* A_3 + i\left(\delta + i\frac{\gamma_2}{2}\right)A_2, \tag{40b}$$

$$\left(\frac{\partial A_3}{\partial t_r}\right)_z = i\Omega_c A_2, \tag{40c}$$

$$\tag{40d}$$

$$\left(\frac{\partial \Omega_p^*}{\partial z}\right)_{t_r} = i\kappa_{12}A_1^* A_2, \tag{41a}$$

$$\left(\frac{\partial \Omega_c^*}{\partial z}\right)_{t_r} = i\kappa_{32}A_3^* A_2. \tag{41b}$$

Here, $A_1, A_2$, and $A_3$ are the same as in Eqs.(2a-2c) except for the removal of the $z$ dependent phase factors. First, recall the example that we have studied before where $\Omega_{c0}\tau = 20$, $\Omega_{p0}\tau = 5$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$. The discussions based on the adiabatic approximation predicts that a population of $|A_3| = 1/\sqrt{17} = 0.242536$ at a depth of $z = 2.86$ for $t_r/\tau > 6.0$, as can be seen from Fig. 2. As a comparison, we show, in Figure 6, a contour plot of $|A_3(z, t_r)|$ for the same parameters given in Figure 2 but using numerical solution of Eqs.(40-41). The flat region in the middle, a region where both of the probe and coupling lasers have been extinguished, indicates that during this period of time the population in the two-photon state remains nearly constant, just as we have concluded before based on the adiabatic theory that a population is left behind in states $|3>$ and $|1>$ which persists for a long time. Correspondingly, we show, in Figure 7, a surface plot of the probe field resulted from the direct numerical solution of Eqs.(40-41) using the same parameters as in Figure 3. We note that there is no probe laser field between $t_r/\tau = 2$ and the time of the injection of the second coupling laser pulse, the same period where the population in the two-photon state remains nearly constant. This indicates, as our adiabatic theory has shown, that photons are neither "stopped" or "stored" in the medium as claimed in Ref. [7-9]. As the original probe photons being slowed down, they are absorbed to produce the two-photon excitation with the above noted population in the state $|3>$. When all probe photons have been absorbed, there will be no further increase of the population in state $3>$. One may still argue that the probe photons are "stored" in the form of atomic coherence involving the two-photon excitation, and later when a second coupling pulse is injected into the medium, the probe pulse is "retrived" from this persist atomic coherence. This is, however, not correct. As we have shown the regenerated pulse has very different time-spectra characteristics from that of the original probe pulse. Therefore, the concept of "retriving the probe pulse" is incorrect. Indeed, the Fourier transform spectrum of all the regenerated fields have shown both different frequency content and noise characteristics that were not there in the original probe pulse. A simple argument based on our result (see Eqs.(37,39)) is sufficient to show that the regenerated pulse is very different from the original probe pulse, that is the regenerated pulse contains information in the second coupling pulse. Therefore, can not be the faithful replica of the original probe pulse alone.

Further comparison between the adiabatic theory and the numerical solution are presented in Figures 8 and 9 for the second set of parameters used to create Figures 4 and 5 which were obtained with adiabatic theory. Finally, we show a case where non-vanishing one-photon detuning is assumed. Figure 10 is surface plot of the probe field obtained from the adiabatic theory with $\delta\tau = 10$, whereas Figure 11 is obtained by directly solving Eqs.(40-41) using the same parameters. All these comparisons show very good agreement between our theory and numerical solutions.

The numerical calculations were also tested by making use of the fact that with $\gamma_2\tau = 0$, the sum of the squares of the three state amplitudes should be unity. In all numerical examples described in this work, this condition of unity was preserved through at least seven significant figures if $\gamma_2\tau = 0$. Also, in all cases when the number of $t_r/\tau$ and $z$ grid points were doubles for all figures shown, the values of the state amplitudes and half-Rabi frequencies repeated through five significant figures at all points where the functions were not near a zero.

## VI. CASE OF FAR FROM RESONANCE

In this Section, we examine the far off resonance excitation, a case has never been studied in literature with the same adiabatic treatment described in previous sections. Specifically, we investigate the condition for $\lambda_0\tau$ to stay well separated from $\lambda_\pm\tau$. We will show analytically that the formulism for far detuned excitation exhibits detuning independent feature when a specific condition is satisfied, a surprising result that to the best of our knowledge has never been reported in the literature.

It is easy to see the condition for the eigenvalues to be well separated when $|\delta| >> |\Omega_c|$ and $|\delta| >> |\Omega_p|$. In this limit

$$\begin{aligned}\sqrt{|\Omega_p|^2 + |\Omega_c|^2 + \left(\frac{\delta + i\gamma_2/2}{2}\right)^2} &\simeq \frac{\delta + i\gamma_2/2}{2}\left(1 + 2\frac{|\Omega_p|^2 + |\Omega_c|^2}{(\delta + i\gamma_2/2)^2}\right) \\ &= \left(\frac{\delta + i\gamma_2/2}{2}\right) + \frac{|\Omega_c|^2 + |\Omega_p|^2}{\delta + i\gamma_2/2}.\end{aligned}$$

The key element that could invalidate the adiabatic treatment is the eigenvalue

$$\lambda_- = -\frac{|\Omega_c|^2 + |\Omega_p|^2}{\delta + i\gamma_2/2} \simeq \frac{|\Omega_c|^2 + |\Omega_p|^2}{\delta},$$

which could become very close to the eigenvalue of the dark state $lamda_0 = 0$ at sufficiently large detuning. Since the length of the pulse due to the coupling field is chosen to be much longer than that due to the probe laser, this eigenvalue can only be well separated from the zero eigenvalue at times when the probe laser first starts to build up if $|\lambda_-\tau| >> 1$ at such times. Physically, what is required is for the ac Stark shift in level $|3>$ to satisfy

$$\frac{|\Omega_c|^2}{\delta}\tau >> 1.$$

This means that as the two-photon transition between $|1>$ and $|3>$ is driven, the final state, i.e. state $|3>$, must be shifted out of exact resonance by an amount that is much larger than the laser bandwidth. (We have assumed that the natural width of level $|3>$ is very small, being due to collisional relaxation in a very cold vapor. If the laser bandwidth is less than the width of the state $|3>$, the latter will replace $\tau^{-1}$ in the above equation.) In Figure 12, we first show a surface plot of $\Omega_p(z,t)$ based on our adiabatic theory for zero detuning. Figure 13 is a corresponding plot where the full numerical evaluations of Eqs.(40-41) are carried out. As expected, the solution based on adiabatic theory agrees well with the numerical solution. We now compare Figures 12 and 13 with Figures 14 and 15 which are the surface plots of $\Omega_p(z,t)$ with large one-photon detuning $\delta\tau = 120$. Figure 14 is based on our adiabatic theory, therefore, should be compared with Figure 12, whereas Figure 15 is the result of full numerical calculation, thus should be compared with Figure 13. From these figures, we therefore conclude that when the appropriate adiabatic condition is met, the solution to the problem is insensitive to the detuning. Indeed, when the conditions given above are satisfied, the adiabatic approximation will be valid for all smaller $\delta$ as well, a surprising results that has not been reported in literature and could play an important role in the future experiments.

## VII. CONCLUSION

In conclusion, we have shown in detail how the adiabatic approximation can be used to understand the propagation of a pair of optical pulses in a highly dispersive medium. Starting from the adiabatic solution of a three-level system interacts with two laser fields, we showed analytically that in the case where the one-photon detuning is small (i.e. $0 \leq \delta < |\Omega_c|$), a coherent optical field with the frequency very close to that of the original probe can be regenerated when a coupling laser pulse in re-injected into the medium at a delayed time. We have provided detailed analysis on the conditions and characteristics of the regenerated field, including the estimate of the pulse width. Our analysis has shown that the regenerated pulse has a very different width therefore cannot be a perfect replica of the original probe pulse. Therefore, the notion of "coherent storage of the original probe pulse" is an incorrect description. As expected, the regenerated pulse is coherent, to an extended degree, with the coupling laser but not the original probe field. Subsequent numerical calculations have confirmed these analytical results. We have further developed the adiabatic theory to the cases where the one-photon detuning is larger enough so that $|\Omega_c| < \delta$ and obtained a different type of adiabatic condition and solution. For time varying probe and coupling fields, the highly nonlinear wave equation for the recurring probe field is then investigated numerically. It is shown that even with this large one-photon detuning, as long as one chooses the pulse envelope and sequence properly so that there is a coherence left in the system, then there will be a coherent optical field generated when the coupling laser is turned on only.

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11. Such a regeneration of an optical field is not unique to the process described in [7-9]. In fact, it is just a stimulated Raman generation in the context of electromagnetically induced transparency. It is not surprising at all that the frequency of the regenerated field is close to that of the original probe field because the second coupling pulse is identical to the first one (in frequency). The regenerated field, however, is not the faithful copy of the probe field as we have shown later. Therefore, the statement of "reviving the probe pulse" in Refs.[7-9] is inaccurate. In fact, one can mix the two lower states by pulsing a magnetic field, and then turning on a coupling laser to regenerate a very similar optical field.


**Figure Captions**

Figure 1. Energy level diagram showing relevant laser excitations. The decay rate of the state $|3>$ is assumed to be very small and hance neglected.

Figure 2. Contour plot of $A_3(z,t)$ based on adiabatic theory. Parameters used: $\Omega_p\tau = 5$, $\Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 11$.

Figure 3. Surface plot of $\Omega_p(z,t)$ based on adiabatic theory. Parameters used: $\Omega_p\tau = 5$, $\Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 11$.

Figure 4. Contour plot of $A_3(z,t)$ based on adiabatic theory. Parameters used: $\Omega_p\tau = \Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 700cm^{-1}$, $R = 3$, $t_d/\tau = 11$.

Figure 5. Surface plot of $\Omega_p(z,t)$ based on adiabatic theory. Parameters used: $\Omega_p\tau = \Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 700cm^{-1}$, $R = 3$, $t_d/\tau = 11$.

Figure 6. Contour plot of $A_3(z,t)$ based on numerical solution of Eqs.(40-41). Parameters used: $\Omega_p\tau = 5$, $\Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 11$. This figure should be compared with the Figure 2.

Figure 7. Surface plot of $\Omega_p(z,t)$ based on numerical solution of Eqs.(40-41). Parameters used: $\Omega_p\tau = 5$, $\Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 11$. This figure should be compared to Figure 3.

Figure 8. Contour plot of $A_3(z,t)$ based on numerical solution of Eqs.(40-41). Parameters used: $\Omega_p\tau = \Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 700cm^{-1}$, $R = 3$, $t_d/\tau = 11$. This figure should be compared with Figure 4.

Figure 9. Surface plot of $\Omega_p(z,t)$ based on numerical solution of Eqs.(40-41). Parameters used: $\Omega_p\tau = \Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 700cm^{-1}$, $R = 3$, $t_d/\tau = 11$. This figure should be compared with Figure 5.

Figure 10. Surface plot of $\Omega_p(z,t)$ based on adiabatic theory with non-vanishing one-photon detuning. Parameters used: $\Omega_p\tau = 10$, $\Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\delta\tau = 10$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 11$.

Figure 11. Surface plot of $\Omega_p(z,t)$ based on numerical solution of Eqs.(40-41) with non-vanishing one-photon detuning. Parameters used: $\Omega_p\tau = 10$, $\Omega_c\tau = 20$, $\gamma_2\tau = 0$, $\delta\tau = 10$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 11$.

Figure 12. Surface plot of $\Omega_p(z,t)$ based on adiabatic theory with zero detuning. Parameters used: $\Omega_p\tau = 10$, $\Omega_c\tau = 40$, $\gamma_2\tau = 0$, $\delta\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 10$.

Figure 13. Surface plot of $\Omega_p(z,t)$ based on numerical evaluation of Eq.s(40-41) with zero detuning. Parameters used: $\Omega_p\tau = 10$, $\Omega_c\tau = 40$, $\gamma_2\tau = 0$, $\delta\tau = 0$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 10$. This figure should be compared with Figure 12.

Figure 14. Surface plot of $\Omega_p(z,t)$ based on adiabatic theory with large one-photon detuning. Parameters used: $\Omega_p\tau = 10$, $\Omega_c\tau = 40$, $\gamma_2\tau = 0$, $\delta\tau = 1200$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 10$. This figure should be compared with Figure 12.

Figure 15. Surface plot of $\Omega_p(z,t)$ based on numerical evaluation of Eq.s(40-41) with large one-photon detuning. Parameters used: $\Omega_p\tau = 10$, $\Omega_c\tau = 40$, $\gamma_2\tau = 0$, $\delta\tau = 120$, $\kappa_{12}\tau = \kappa_{32}\tau = 200cm^{-1}$, $R = 4$, $t_d/\tau = 10$. This figure should be compared with Figure 13.

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